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The sectional strain ellipse during progressive coaxial deformations

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Abstract—The two-dimensional strain history on a sheet which is inclined to the principal axes of the strain ellipsoid is considered. Even if the strain history in three dimensions is coaxial, the two-dimensional progressive deformation on the surface of the sheet is in general of a non-coaxial type. It is shown in this paper that the degree and sense of two-dimensional non-coaxiality is governed by the strain path followed during three-dimensional coaxial deformation. The general relationship is defined between the gradient of the strain path on the Flinn strain ellipsoid diagram and the nature of the two-dimensional strain increments. For most strain paths an asymmetrical arrangement of structures in the oblique sheet is to be expected. Hence, en échelon folds, transected folds and extension veins with curved fibres could be produced during three-dimensional coaxial deformation. Only if the strain path is of a rather special type will the deformation be coaxial in a two-dimensional as well as a three-dimensional sense.

INTRODUCTION

'COAXIAL' and 'non-coaxial' are terms which describe the position of the axes of the strain ellipsoid with respect to lines defined by material points during progressive deformation (Hsu 1966, Elliott 1972, Hobbs *et al.* 1976). A coaxial strain history is one which involves the coincidence of the principal axes of strain and the same set of material lines throughout the strain history. The principal axes of strain rate (incremental strain) and finite strain are always parallel during a coaxial strain history. In the study of structures whose characteristics are determined solely by the state of finite strain, the distinction between coaxial and non-coaxial becomes irrelevant. However, many structures and rock fabrics appear to inherit some of their features from intermediate stages in the deformation history and for these the presence and nature of the strain non-coaxiality becomes important.

The definition above is a three-dimensional one as it refers to the strain ellipsoid. The indications we obtain about the nature of the strain history however are very often based on two-dimensional observations on planar outcrop surfaces, thin sections or geological surfaces. In order to be able to draw three-dimensional conclusions from such two-dimensional data we need to investigate the behaviour of the strain ellipse of the plane on which our observations are made. For instance, does this ellipse, referred to below as the sectional strain ellipse, experience a two-dimensional coaxial strain history when the rock is undergoing a coaxial strain in the three-dimensional sense? This paper is concerned with questions of this nature and examines the development of the sectional strain on a plane inclined to the principal axes of the strain ellipsoid during progressive deformation. The discussion, which is restricted to three-dimensional coaxial strains, allows one to formulate rules which apply when the coaxial strain concept is extended to consider the strains which develop in an oblique planar section.

The question of the strains which develop on a general section through a deforming rock has been considered by several authors (e.g. Flinn 1962, Ramsay 1967, p. 175, Treagus & Treagus 1981). These authors point out that the relationship between the sectional and three-dimensional strain can be complex even when the latter is of a coaxial type. It will be shown below, however, that the non-coaxial character of the sectional strain can be readily predicted from the nature of the strain path followed during a coaxial strain history.

The V *property*

Fundamental to the discussion of orientation of the principal axes of the ellipse produced by sectioning an ellipsoid is the V property. This is equivalent to the $2V$ parameter much utilized in optics to determine the orientation of ellipses produced by sectioning the optical indicatrix ellipsoid. Flinn (1962) has pointed out the importance of this parameter for the strain ellipsoid when the principal axes of the sectional strain ellipse are to be determined.

The angle V describes the orientation of the circular sections of the ellipsoid with respect to its principal axes. The V angle described in the discussion that follows is defined as the angle between the λ_1 axis and the normal to the circular section. The V angle defined in this way (V_{λ_1}) is 0° for a uniaxial prolate ellipsoid and 90° for a uniaxial oblate ellipsoid. By considering these end-member ellipsoids we can directly anticipate the way the V angle controls the orientation of the sectional strain ellipse. Any section through a uniaxial prolate ellipsoid produces an ellipse whose long axis is the orthogonal projection of the λ_1 principal axis of the ellipsoid on the section plane. A uniaxial oblate ellipsoid on the other

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hand always sections to yield an ellipse with a long axis parallel to the line of intersection of the section plane and the $\lambda_1\lambda_2$ (circular) plane of the strain ellipsoid. In other words, if the shape of the ellipsoid changes so that V increases, the long axis of the sectional ellipse moves away from the projection of λ_1 towards the intersection line with the $\lambda_1\lambda_2$ plane.

A simple graphical construction, the Fresnel construction, exists for the direction of the principal axes of the sectional ellipse. This construction is described in Wahlstrom (1951, p. 232) and Lisle (1976). For a given orientation of the ellipsoid and section plane, the Fresnel graphical construction allows calculation of the orientation of the axis of the sectional ellipse if the angle V of the ellipsoid is known. The latter can be calculated from the reciprocal quadratic elongations ($\lambda' = 1/\lambda$)

$$\tan^2 V = \frac{\lambda_3' - \lambda_2'}{\lambda_2' - \lambda_1'} \qquad (1)$$

or from the axial ratios of the strain ellipsoid, $a = (\lambda_1/\lambda_2)^{1/2}$ and $b = (\lambda_2/\lambda_3)^{1/2}$

$$\tan^2 V = \frac{a^2(b^2 - 1)}{a^2 - 1} \qquad (2)$$

from Flinn (1962).

Equations for the orientation of the principal axes of the sectional ellipse have been given recently by a number of authors (Ramberg 1976, Ferguson 1979, Fry 1979, Gendzwill & Stauffer 1981, Treagus & Treagus 1981). These equations allow the calculation of the pitch of the long axis of the ellipse produced by sectioning an ellipsoid of known axial lengths on a plane of known orientation with respect to the principal axes of the ellipsoid. However, the existence of the Fresnel construction implies that a knowledge of the individual lengths of the ellipsoid axes is unnecessary for this purpose. The required relationship can be derived from the equations of Treagus & Treagus (1981, appendix II, eqns 18, 19 and 25). Consider an inclined plane (Fig. 1) whose orientation relative to the principal axes of the strain ellipsoid is given by strike (α) and dip (β). Treagus & Treagus (1981, eqn 25) showed that the pitch (θ) of the long axis of the sectional strain ellipse is given by

$$\tan 2\theta = \frac{2(\lambda_3' - \lambda_2')\sin\alpha(1 - \sin^2\alpha)^{1/2}}{\lambda_D' - \lambda_S'}, \qquad (3)$$

where λ_D' and λ_S', the reciprocal quadratic elongations of lines within the plane parallel to the lines of dip and strike, respectively, are calculated from eqns 18 and 19 of Treagus & Treagus (1981)

$$\lambda_S' = (1 - \sin^2\alpha)\lambda_2' + \sin^2\alpha\lambda_3' \qquad (4)$$

$$\lambda_D' = \sin^2\beta\lambda_1' + \sin^2\alpha(1 - \sin^2\beta)\lambda_2' + (1 - \sin^2\alpha)(1 - \sin^2\beta)\lambda_3'. \qquad (5)$$

Substitution for λ_D' and λ_S' in eqn (3) and inverting yields

$$\cot 2\theta = \frac{(\lambda_1' - \lambda_3')\sin^2\beta + (\lambda_3' - \lambda_2')(1 - 2\sin^2\alpha + \sin^2\alpha\sin^2\beta)}{2(\lambda_3' - \lambda_2')\sin\alpha(1 - \sin^2\alpha)}$$

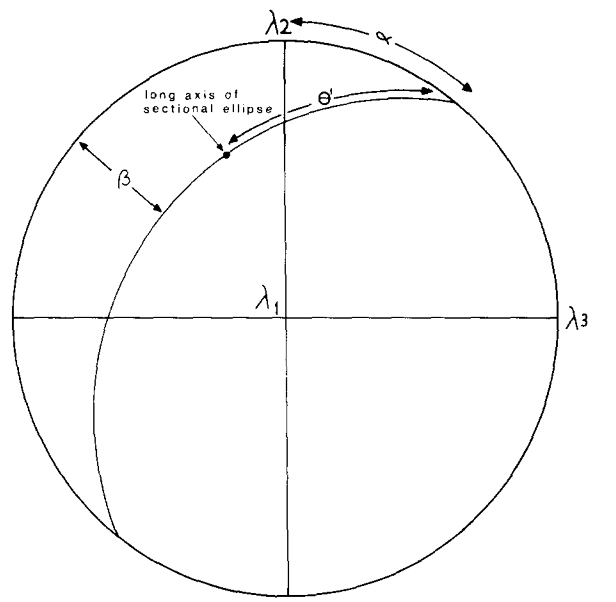

Fig. 1. The angles used to describe the orientation, in the deformed state of the strain ellipse on a plane inclined to the principal axes of the strain ellipsoid. See text for details.

which, after combination with (1) and some manipulation, is

$$\tan 2\theta = \frac{\sin 2\alpha}{\cos 2\alpha - \sin^2\beta(\cot^2 V + \cos^2\alpha)}. \qquad (6)$$

In this equation the V value of the ellipsoid alone allows the orientation of the sectional ellipse to be calculated for a plane of given orientation.

PROGRESSIVE DEFORMATION AND TWO TYPES OF SECTIONAL STRAIN

I will consider below the behaviour of the sectional strain ellipse on a fixed plane in a rock undergoing a three-dimensional coaxial strain history. Before this can be done we must analyse more closely what is meant by 'fixed plane'. Two meanings suggest themselves. In the first, the plane is fixed in orientation with respect to the axes of the strain ellipsoid. In the other, the plane is fixed in position within the material. The latter can be referred to as a 'material plane'.

Sectional strain on a plane fixed in orientation with respect to the axes of the strain ellipsoid

The orientation of the strain ellipse on this type of fixed plane will change in a way governed by the way the V parameter of the ellipsoid changes during the strain history. A Flinn diagram like that shown in Fig. 2 can be used to represent strain paths which record the progressive changes of axial ratios undergone by the strain ellipsoid during the strain history. Also shown on Fig. 2 are V curves; that is curves linking all ellipsoids which share the same value of V. Clearly, if a strain path proceeds along such a V curve, the sectional ellipse will retain a constant orientation with respect to the principal

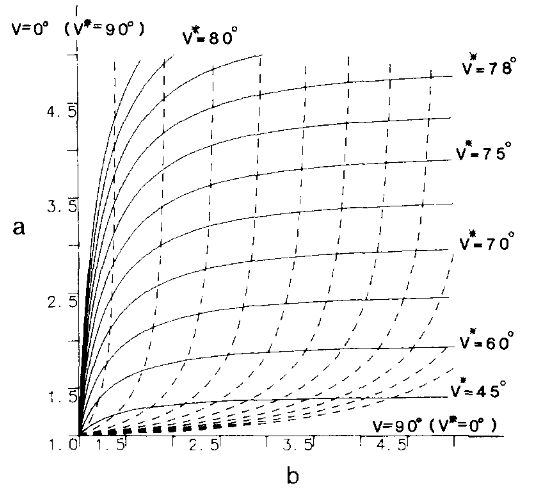

Fig. 2. Flinn graph ($a = (\lambda_1/\lambda_2)^{1/2}$, $b = (\lambda_2/\lambda_3)^{1/2}$) showing curves of constant V for the strain ellipsoid (dashed lines) and curves of constant V for the reciprocal strain ellipsoid (solid lines).

axes of the strain ellipsoid (the sectional strain can be referred to as *irrotational*). All other strain paths will intersect the V curves and yield *rotational* strain ellipse histories. Because it is difficult to envisage a geological situation in which such a two-dimensional strain history could be observed we will proceed to our second type of sectional strain.

Sectional strain on a material plane

The section plane considered here in general rotates with respect to the principal axes of the strain ellipsoid so the V parameter of the strain ellipsoids during deformation tells us little about the state of strain on this section plane (i.e. the angles α and β in eqn 6 change during deformation). Instead, reference to the reciprocal strain ellipsoid and its corresponding V parameter proves to be more useful.

The reciprocal strain ellipsoid (e.g. Jaeger 1962) can be thought of as the shape before deformation which deforms homogeneously to give a sphere. As such, it is a description of the strain referred to the undeformed state. The reciprocal strain ellipse, a section through the reciprocal strain ellipsoid, portrays inversely the variable extensions to be suffered by lines within that plane. For example, the material line coinciding with the short axis of the reciprocal strain ellipse will be the same material line which is parallel to the long axis of the strain ellipse on the deformed plane.

The V value of the reciprocal strain ellipsoid, which we denote V^*, allows us to calculate, for a material plane *in the undeformed state*, the material line which is to become the line with maximum elongation in that material plane, in the deformed state. Clearly, if, during the progressive deformation, the reciprocal strain ellipsoid retains the same V^* value then it is the same material line in the undeformed state which becomes at every stage in the deformation history the long axis of the strain ellipse in that material plane in the deformed state. Such a deformation history (a V^* strain path) leads therefore to a coaxial sectional strain history for all

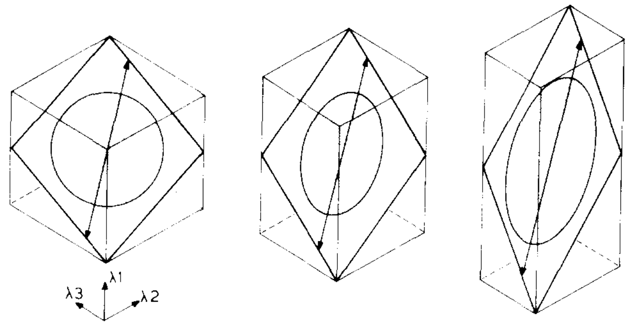

Fig. 3. Stages in a coaxial sectional strain history. The long axis of the strain ellipse on the inclined material plane remains associated with the same material line during the progressive deformation.

material planes. Figure 3 illustrates a coaxial sectional strain history. Using symbols with an asterisk for the reciprocal strain ellipsoid we have

$$\lambda_1'^* = \lambda_3$$
$$\lambda_2'^* = \lambda_2$$
$$\lambda_3'^* = \lambda_1$$
$$a^* = b$$
$$b^* = a$$

and, substituting into eqn (1), V^* (for the reciprocal strain ellipsoid) becomes

$$\tan^2 V^* = \frac{\lambda_1 - \lambda_2}{\lambda_2 - \lambda_3} = \frac{b^2(a^2 - 1)}{b_2 - 1}. \qquad (7)$$

Lines of constant V^* are shown in Fig. 2. These curves allow us to decide if a plotted strain path will give rise to coaxial or non-coaxial sectional strain. Strain paths which intersect the constant V^* curves will produce sectional non-coaxial strains whilst strain paths following these curves will correspond to coaxial sectional strain histories. It becomes apparent that coaxial strain in a two-dimensional as well as a three-dimensional sense is a very special type of strain history.

The important deduction that the strain will be coaxial when the strain path follows a V^* curve on the Flinn diagram is demonstrated mathematically below with reference to the shape characteristics of the finite and infinitesimal strain ellipsoids during a coaxial strain increment.

INFINITESIMAL AND FINITE STRAIN ELLIPSOIDS

By definition a coaxial strain history requires that the axes of the principal axes of the ellipsoid representing the last small increment of strain (the infinitesimal strain ellipsoid) must be parallel to the axes of the ellipsoid recording the total accumulated strains (the finite strain ellipsoid). Since this paper discusses only three-dimensional strain histories which are of a coaxial type, this condition must apply here.

It has been reasoned in the previous section that the two-dimensional strain histories of all planes through the deforming rock will be coaxial if the strain path is such that V^* retains a constant value. When this is the

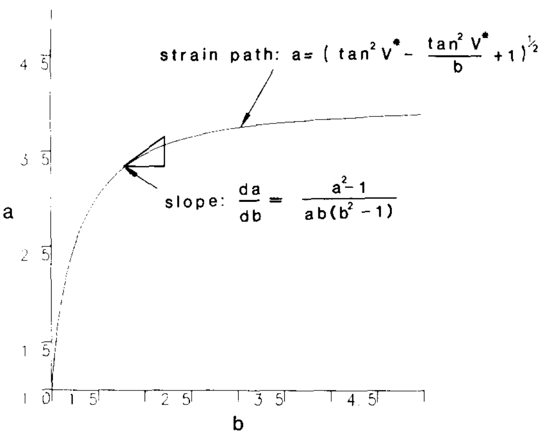

Fig. 4. A V^* strain path.

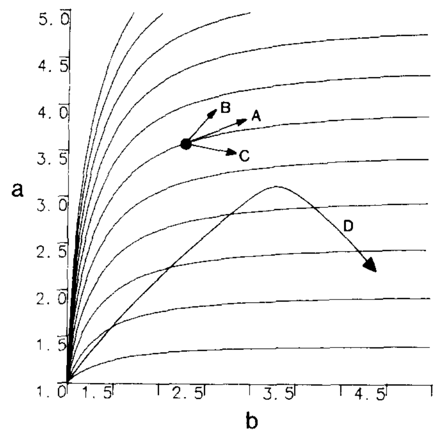

Fig. 5. Strain paths in relation to V curves. A, B and C are strain path segments with various slopes with respect to the slope of the V^* curves. A will give rise to a coaxial strain increment, B and C to positive and negative non-coaxial, respectively. D is a complex strain path and involves increments of the types A, B and C.

case, the principal axes of the finite strain ellipse must be parallel to those of the infinitesimal strain ellipse. Considering a given plane of strike α and dip β, the pitches of the long axes of these ellipses (θ_f, θ_i) are given by (6). It would appear then from eqn (6) that under these conditions, $V_i = V_f$. This is proved below.

From (7), the equation of the V^* strain path (Fig. 4) can be found to be

$$a = \left(\tan^2 V^* - \frac{\tan^2 V^*}{b^2} + 1\right)^{1/2} \tag{8}$$

and its slope

$$\left(\frac{da}{db}\right)_{V^*} = \left(\frac{a^2 - 1}{ab(b^2 - 1)}\right)_{V^*}. \tag{9}$$

The slope of any strain path on the Flinn plot can be shown to be related to the k value ($k = (a - 1)/(b - 1)$) of the infinitesimal strain (k_i) at that point on the path by the equation

$$\frac{da}{db} = k_i \frac{a}{b}. \tag{10}$$

Combining (9) and (10) gives

$$(k_i)_{V^*} = \left(\frac{a^2 - 1}{a^2(b^2 - 1)}\right)_{V^*}. \tag{11}$$

For the infinitesimal strain ellipsoid the k value and V angle are simply related (Flinn 1962, p. 389). The equation relating them is derived from (2) considering the condition of small strains (ab approaches unity, $b \rightarrow 1/a$)

$$\tan^2 V_i = \frac{1}{k_i} \tag{12}$$

which, when combined with (11), gives

$$\tan^2 (V_i)_{V^*} = \left(\frac{a^2(b^2 - 1)}{a^2 - 1}\right)_{V^*}. \tag{13}$$

But from (2)

$$\tan^2 V_f = \frac{a^2(b^2 - 1)}{a^2 - 1}. \tag{14}$$

Thus it is proved, that under conditions of constant V^*,

$$V_i = V_f$$

and hence the sectional strain history is of coaxial type.

Figure 2 shows that the V value of the finite ellipsoid following a V^* path must constantly increase as the strain magnitude increases. To maintain its course on a V^* path, the infinitesimal strains must therefore change their V values continuously to keep pace with the V of the finite strain ellipsoid. A non-coaxial sectional strain history results when the strain path deviates from a V^* path implying an inequality between the V values of the infinitesimal and finite strain ellipsoid.

Figure 5 shows portions of three strain paths. If the slope of the strain path is parallel to the V^* curves (path A) an increment has been coaxially added to the sectional strain ellipse. Strain path B in Fig. 5 has a steeper slope than the V^* curve which means that $V_i < (V_i)_{V^*}$. Also, $V_i < (V_f)_{V^*}$ because $(V_i)_{V^*} = (V_f)_{V^*}$. The superimposed increments in other words are more prolate than the finite strain and the orientation of the long axis of the incremental ellipse lies closer to the orthogonal projection of the λ_1 axis on the considered plane than does the long axis of the finite strain ellipse. I define this sense of non-coaxiality as positive. The terms positive and negative non-coaxiality are illustrated in Fig. 6. Path C (Fig. 5) with a lower slope than the intersecting V^* curve will give rise to a negative non-coaxiality.

THE SECTIONAL STRAIN HISTORY ASSOCIATED WITH THE SIMPLEST OF STRAIN PATHS

On the basis of an observed strain path, we are now in a position to deduce the presence and its sense of non-coaxiality in the section strain history. I consider now a specific example. The simplest coaxial strain path arising from the superimposition of identical strain increments leads to strain path of constant K, where $K = \ln a/\ln b$ (Ramsay 1967, p. 329). As can be seen in Fig. 7, K strain paths always possess a steeper slope than V^*

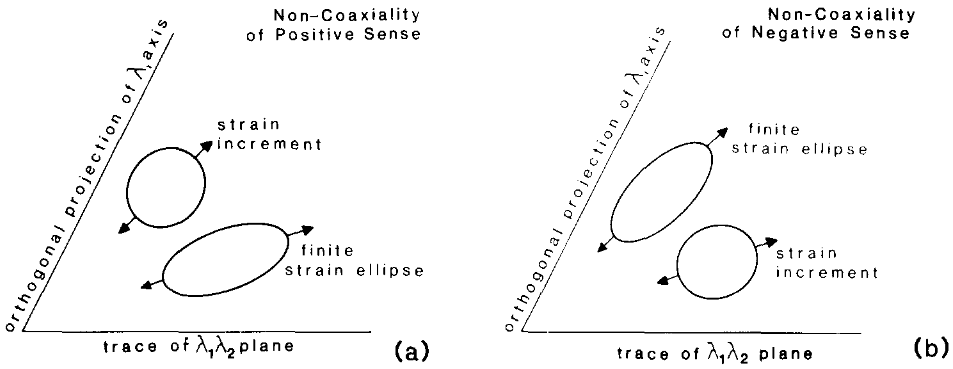

Fig. 6. (a) Non-coaxial section strain of a positive sense where the finite strain ellipse is oriented closer to the $\lambda_1\lambda_2$ plane trace than the ellipse representing the last strain increment. (b) Negative non-coaxiality is where the incremental strain ellipse is closer to the $\lambda_1\lambda_2$ plane trace than the finite strain ellipse.

curves. This means, for the reasons explained in the previous section, that the sectional non-coaxiality of non-principal planes will be positive. The sense of non-coaxiality undergoes no reversals but remains positive throughout the deformation history.

THE MAXIMUM LIMIT ON THE SECTIONAL NON-COAXIALITY DURING A THREE-DIMENSIONAL COAXIAL STRAIN HISTORY

The measure of non-coaxiality used in this discussion is an instantaneous one defined by angular deviation between the finite strain ellipse and the incremental strain ellipse at that instant (see Elliott 1972 and Means *et al.* 1980 for other measures of non-coaxiality). The amount of non-coaxiality, expressed as the angle ($\theta_f - \theta_i$), is a function of the attitude of the section plane with respect to the three principal axes of the strain ellipsoid and also of the angles V_i and V_f. It can be evaluated by means of eqn (6) noting, in relation to the strain path in Flinn space (a = function (b)), that

$$\cot^2 V_i = \frac{da}{db}\cdot\frac{b}{a} \quad \text{and} \quad \cot^2 V_f = \frac{a^2 - 1}{a^2(b^2 - 1)}.$$

An upper limit is placed on the amount of non-coaxiality by the fact that this angle of obliquity between the ellipses can never exceed the angle between the traces of the $\lambda_1\lambda_2$ plane and the projection of the λ_1 axis on the section plane. The value of this maximum limit therefore depends strongly on the inclination of the plane to the principal axes of the strain ellipsoid. Figure 8 illustrates how the plane's orientation constrains the maximum permissible angular non-coaxiality. Planes parallel to the principal axes, for instance, can never show non-coaxiality. Marked non-coaxiality results when the normal to the plane lies close to the $\lambda_1\lambda_3$ plane (i.e. when the plane lies close to λ_2 axis of the strain ellipsoid). These established theoretical limits on the amount of non-coaxiality can be exceeded only if, at a given point, the directions of λ_1, λ_2 and λ_3 of the infinitesimal strain ellipsoid become interchanged.

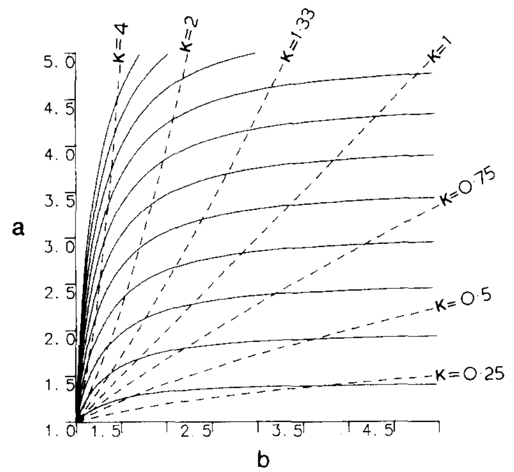

Fig. 7. Flinn diagram comparing the shape of K curves with V curves. Simple strain paths of constant K will be characterized by positive non-coaxiality of the sectional strain because of the steeper gradient of the K-curves compared with the V^* curves.

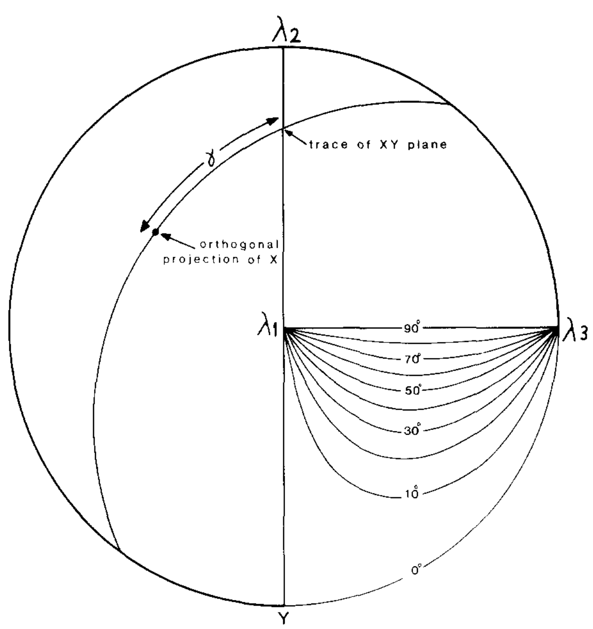

Fig. 8. Stereographic projection showing how γ, the angle in the section between the trace of the $\lambda_1\lambda_2$ plane and the orthogonal projection of λ_1, varies as a function of the plane orientation. The lower right quadrant shows how the magnitude of γ varies as a function of the orientation of the normal to the section plane.

GEOLOGICAL PHENOMENA RELATED TO THE NON-COAXIALITY OF SECTIONAL STRAIN

Models for the rotation of fold axes during progressive deformation

Ramberg (1959) and Flinn (1962) have made the suggestion that embryonic folds that develop in a layer oblique to the principal axes of the strain will have axes perpendicular to the direction of maximum finite shortening within that layer.

Two theories exist for the subsequent rotational behaviour of the fold axes. Flinn (1962), Sanderson (1973) and Ramsay (1979) considered that folds rotate during continued strain as passive (material) lines whereas Osberg (1975), Stringer & Treagus (1980) and Treagus & Treagus (1981) envisaged that folds rotate so as to remain perpendicular to the direction of maximum finite shortening within the folding sheet.

The relationship between these two models of hinge rotation has clearly to do with the non-coaxiality of the sectional strain. The models become equivalent when the three-dimensional strain history is one of the special cases discussed above which produces a coaxial sectional strain history. The second model mentioned above does not therefore necessarily imply that fold axes always behave as non-material lines. Treagus & Treagus (1981) did not recognize this since they considered only constant-K strain paths.

Folds rotating in the manner proposed in the second model will rotate towards the λ_1 direction at a rate faster than that of a material line in the case of a positive non-coaxiality and slower than a material line when the non-coaxiality of the sectional strain is negative. Non-coaxiality will lead to hinge migration with respect to material points in the folding layer. The non-coaxiality of the strain within a sheet undergoing folding could result in an asymmetrical arrangement of structures within the sheet (Fig. 9). It has been suggested that en échelon folds (Ramsay 1967, p. 177, Treagus & Treagus 1981) and folds with transecting cleavage (Borradaile 1978, Stringer & Treagus 1980, Treagus & Treagus 1981) are possible products of this asymmetry and that they may provide evidence of the sense of non-coaxiality of the strain history in the plane of the folding sheet.

Structures related to incremental extensions

Ramsay (1967), Elliott & Wickham (1970) and Durney & Ramsay (1973) have interpreted the curved pattern shown by crystals of fibrous habit present in veins or separation zones between boudins in terms of the incremental history of dilation. The discussion above leads us to conclude that the changing direction of maximum extension indicated by curved fibres in two dimensions does not necessarily imply a non-coaxial strain in three dimensions, but should also be a common feature of three-dimensional coaxial history where the structure is developed in extending layers oblique to the principal axes of the ellipsoid.

Complexities in the shape of these fibres might be explained in terms of the strain path and its relation to the V^* paths. For instance the presence of inflection points in curved fibres might indicate a stage in the sectional strain history corresponding to a coaxial sectional increment which marks the reversal of sense of sectional non-coaxiality [Figs. 5 (path D), 9 and 10]. Along a constant-K strain path curved fibres will be produced though inflections will not arise because the sense of non-coaxiality remains constant (and positive) throughout.

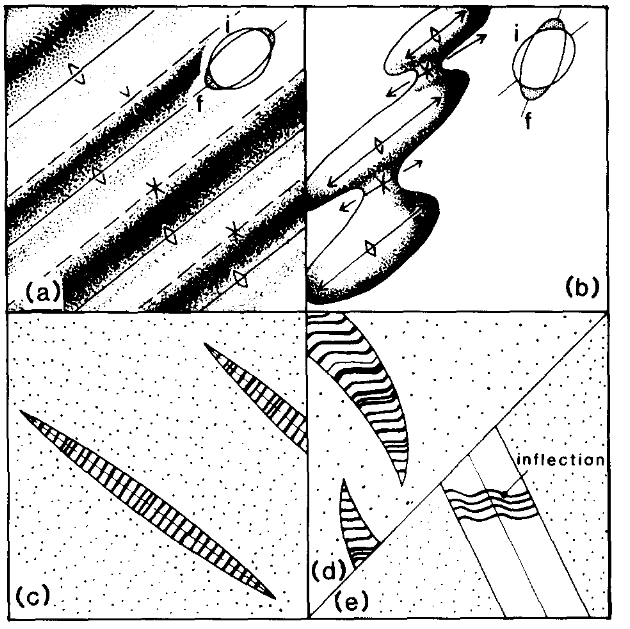

Fig. 9. Summary diagram of structures produced by sectional coaxial (a and c) and non-coaxial (b, d and e) strain. (a) and (b) represent folding while (c), (d) and (e) show extension-vein structures. Finite (f) and incremental (i) strain ellipses are shown.

CONCLUSIONS

This paper emphasizes that coaxial strain in a three-dimensional sense is not necessarily accompanied by coaxial strain on planar sections through the rock. In fact, non-coaxial sectional strain is to be expected as a general outcome of three-dimensional coaxial strains. Conversely, some three-dimensional strain histories such as simple shear do not automatically give rise to non-coaxial sectional strains. These relationships are summarized in Table 1.

Table 1. Summary scheme: coaxial and non-coaxial strain in two- and three-dimensions

	Sectional strain history	
	Coaxial	Non-coaxial
3-D Coaxial strain history	Special. Requires specific strain path otherwise exists on principal planes only	General situation for 3-D coaxial strains
3-D Non-coaxial strain history	Rare but possible, e.g. on planes parallel to λ_2 in simple shear strain history	The most probable situation in nature

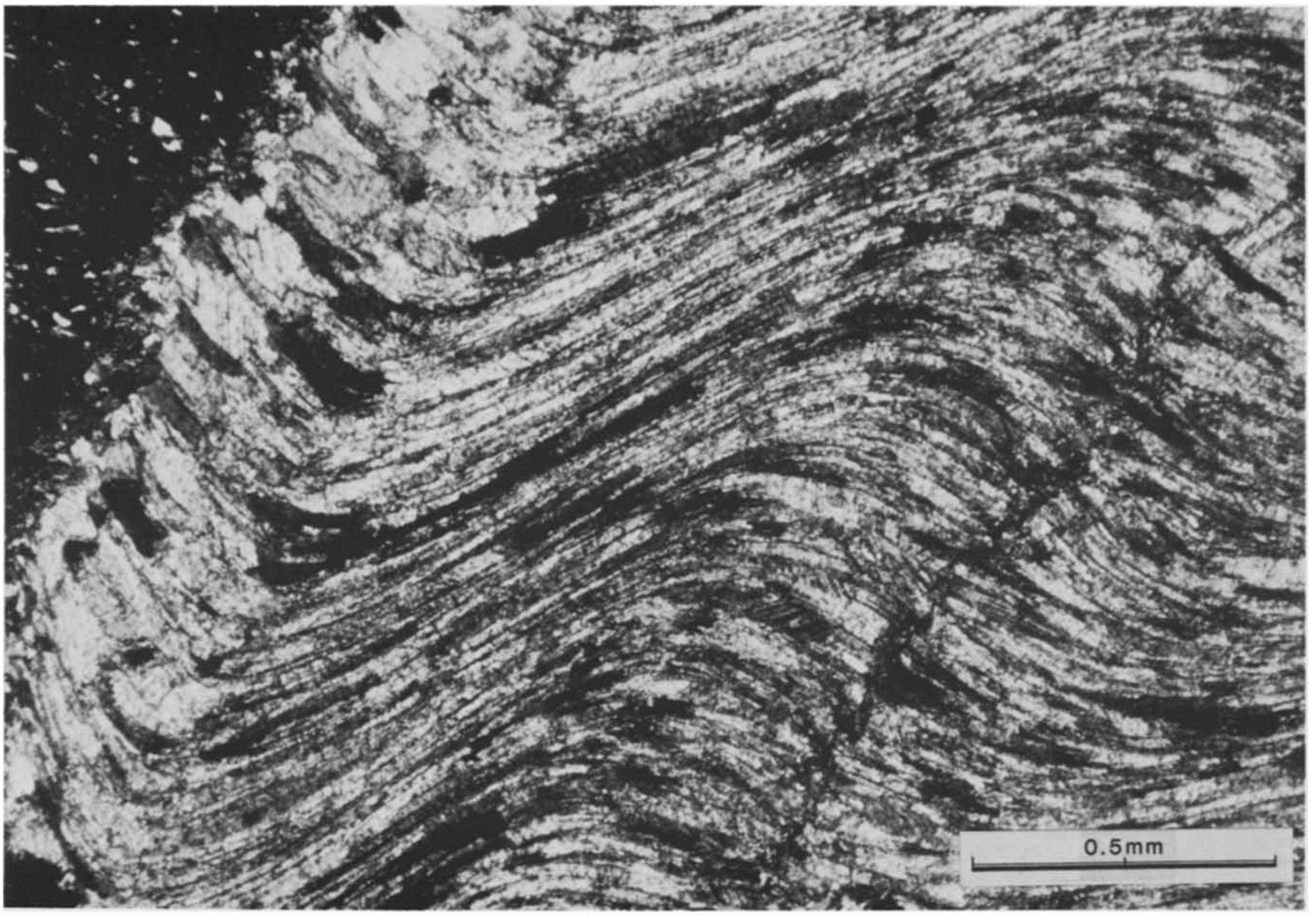

Fig. 10. Curved calcite fibres in a vein, Permian Slates, Bielsa, Southern Pyrenees. The presence of an inflection point can be used to indicate a strongly curved strain path as shown in Fig. 5 [curve D].

Only if the strain path is of a rather special type will the strain history be truly coaxial in a two-dimensional as well as a three-dimensional sense. Although there is no reason to expect that such special strain paths are common geologically, they are nevertheless important as they give rise to strains which mark the transition between positive and negative sectional non-coaxiality sense. The presence and sense of non-coaxiality established from incremental-strain structural indicators can be used to estimate the slope of the strain path at various stages in progressive deformation if the deformation is coaxial in three dimensions. In general, pronounced sectional non-coaxiality will result from strong curvatures of the strain path.

Finally, it should be emphasized that this discussion has been totally restricted to the case of three-dimensional strain histories which are coaxial. The sectional coaxial character of such histories is in general observable only on planes containing a principal axis of the finite strain ellipsoid. Caution should be exercised before conclusions are drawn concerning the non-coaxiality of three-dimensional strain from observations of structures visible on non-principal planes through the deformed rock.

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REFERENCES

Borradaile, G. J. 1978. Transected folds: a study illustrated with examples from Canada and Scotland. *Bull. geol. Soc. Am.* **89**, 481–493.

Durney, D. W. & Ramsay, J. G. 1973. Incremental strains measured by syntectonic crystal growths. In: *Gravity and Tectonics* (edited by De Jong, K. A. & Scholten, R.). Wiley, New York, 67–96.

Elliott, D. 1972. Deformation paths in structural geology. *Bull. geol. Soc. Am.* **72**, 1595–1620.

Ferguson, C. C. 1979. Intersection of ellipsoids and planes of arbitrary orientation and position. *Math. Geol.* **11**, 329–336.

Flinn, D. 1962. On folding during three-dimensional progressive deformation. *Q. J. geol. Soc. Lond.* **118**, 385–433.

Fry, N. 1979. Density distribution techniques and strained length methods for the determination of finite strain. *J. Struct. Geol.* **1**, 221–229.

Gendzwill, D. J. & Stauffer, M. R. 1981. Analysis of triaxial ellipsoids: their shapes, plane sections and plane projections. *Math. Geol.* **13**, 135–152.

Hobbs, B. E., Means, W. D. & Williams, P. F. 1976. *An Outline of Structural Geology*. Wiley, New York.

Hsu, T. C. 1966. The characteristics of coaxial and non-coaxial strain paths. *J. Strain Analysis* **1**, 216–222.

Jaeger, J. C. 1962. *Elasticity, Fracture and Flow*. Methuen, London.

Lisle, R. J. 1976. Some macroscopic methods of fabric analysis. *J. Geol.* **84**, 225–235.

Means, W. D., Hobbs, B. E., Lister, G. S. & Williams, P. F. 1980. Vorticity and non-coaxiality in progressive deformations. *J. Struct. Geol.* **2**, 371–378.

Osberg, P. S. 1975. Minor folds and derived orientations of strain in the east flank of the Berkshire Massif, Massachusetts. *Prof. Pap. U.S. geol. Survey* **888**, 97–103.

Ramberg, H. 1959. Evolution of ptygmatic folding. *Norsk. geol. Tidsskr.* **39**, 99–151.

Ramberg, H. 1976. The strain in a sheet intersecting the strain ellipsoid at any angle. *Bull. Soc. geol. Fr.* **18**, 1417–1422.

Ramsay, D. M. 1979. Analysis of rotation of folds during progressive deformation. *Bull. geol. Soc. Am.* **90**, 732–738.

Ramsay, J. G. 1967. *Folding and Fracturing of Rocks*. McGraw-Hill, New York.

Stringer, P. & Treagus, J. E. 1980. Non axial planar S_1 cleavage in the Hawick Rocks of the Galloway Area, Southern Uplands, Scotland. *J. Struct. Geol.* **2**, 317–331.

Sanderson, D. J. 1973. The development of fold axes oblique to the regional trend. *Tectonophysics* **16**, 55–70.

Treagus, J. E. & Treagus, S. H. 1981. Folds and the strain ellipsoid: a general model. *J. Struct. Geol.* **3**, 1–17.

Wahlstrom, E. E. 1951. *Optical Crystallography* (2nd Edn). Wiley, New York.

Wickham, J. S. & Elliott, D. 1970. Rotation and strain history in folded carbonates, Front Royal area, northern Virginia. *Trans. Am. geophys. Un.* **51**, 422.